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Nonlinear Wave solutions of the β-Fractional Burgers–Huxley Equation via Modified Extended Direct Algebraic Method

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Abstract
In this paper, we study nonlinear wave solutions of the β-fractional Burgers-Huxley (FBH) equation in the sense of the Beta derivative. The modified extended direct algebraic method (MEDAM) is utilized to obtain the exact analytical traveling-wave solutions for the nonlinear fractional model. By adopting a suitable traveling-wave transformation, the fractional PDE is transformed into an ODE and explicit analytical solutions can be obtained. For different values of the parameters, a number of solution families, namely hyperbolic, trigonometric and rational type solutions, are generated and plotted. It is found from the analysis that the changes of the fractional order have significant effect on the amplitude and the wave localization. The results demonstrate the effectiveness and generality of the MEDAM method to deal with nonlinear fractional PDEs and disclose more interesting wave phenomena in nonlinear systems.

Keywords: Keywords: Fractional Burgers-Huxley equation; traveling wave transformation; Beta fractional derivative; modified extended direct algebraic method.

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1. Introduction:

Nonlinear partial differential equations (NlPDEs) are the central instruments for the description of complicated nonlinear processes observed in science and technology. Nonlinear physical processes such as diffusion transport, reaction kinetics, solid-state reactions, plasma physics, fluid mechanics, waves, and capillary–gravity wave motion can be effectively modelled using NPDE equations. These formulas are commonly applied to fields like engineering, financial modeling, fluid dynamics and physical chemistry, where nonlinear relationships define the behavior of the system in question. These couplets sometimes result in a menagerie of time-dependent phenomena such as soliton propagation, wave modulation and stability, pattern formation and space–time structures in a number of physical contexts [1–9]. Hence, in order to describe and analyze the fundamental nonlinear dynamics, it is necessary for analytical solutions of partial differential equations to be obtained. Several scientists have been introduced different analytic and numerical methods for finding the exact solution of NLPDEs and nonlinear fractional partial differential equations (NLFPDEs).The Kudryashov method [10], Lie group method [11], generalized Riccati equation mapping [12], Riccati–Bernoulli sub-ode method [13], Improved modified extended tanh function method [14], Extended hyperbolic function method [15], Extended *(G'/G)*-expansion method [16], and He's homotopy perturbation method [17]. Recently, the study of solution for the fractional PDEs (FPDEs) has been developed rapidly by using different approached including Beta fractional derivative, conformable fractional derivatives, Truncated M-Fractional Derivative Caputo fractional derivatives, etc. ([18]-[25]).

The Burgers-Huxley (BH) equation is one of the key nonlinear reaction-convection–diffusion equation which is obtained by merging the classical Burgers convection-diffusion model that has been originally proposed by Bateman [26], later on studied by Burgers [27], with the nonlinear reaction described by the Hodgkin-Huxley model [28]. The BH expresses the balance between diffusion and convection mechanisms of fluid flow and the reaction–diffusion features of the Huxley equation which describes the dynamics of nerve impulse propagation in neurophysics and the flame front propagation in patterned liquids which thus ends up as a crucial equation in nonlinear wave propagation in biological, chemical, physical, financial, population dynamical systems [29]. The Generalized BH is defined by [29]:

$$p_t - v p_{xx} + \varrho p p_x = \kappa \mathrm{p}(\mu - \mathrm{p}^{\eta})(\mathrm{p}^{\eta} - 1), \tag{1}$$

where *p(x,t)* is a scalar field that is a function of the spatial variable *x* and the temporal variable *t*, and $\boldsymbol{v}$,ϱ,$\boldsymbol{v}$,μ and η are constants. The term $\varrho p p_x$ characterizes the nonlinear convection effect, $\boldsymbol{v} p_{xx}$ represents the diffusion mechanism, and $\kappa p(\mu - p^{\eta})(p^{\eta} - 1)$ relates to the reaction kinetics. For the parameter values $\eta = \kappa = \boldsymbol{v} = 1$, $\varrho = -1$, Eq. (1) is reduced to the famous BH equation [31].

$$p_t - p_{xx} - p p_x - \mathrm{p}(\mu - \mathrm{p})(\mathrm{p} - 1) = 0. \tag{2}$$

The fractional Burgers–Huxley equation (FBH) equation, which is defined as:

$${}_0^A D_t^{\beta} p - {}_0^A D_{xx}^{\beta} \mathrm{p} - p\, {}_0^A D_x^{\beta} p - \mathrm{p}(\mu - \mathrm{p})(\mathrm{p} - 1) = 0, \tag{3}$$

where ${}_0^A D_t^{\beta}$ and ${}_0^A D_x^{\beta}$ represent the BFD of order β∈(0, 1] with respect to time *t* and space *x* respectively, and ${}_0^A D_{xx}^{\beta}$ is second BFD with respect to space *x*.

In this work, we concentrate on two-dimensional line-shaped wave solutions of Eq. (3) using BFD. The BFD, pioneered by [18], are a path-breaking development in the realm of fractional calculus. These operators have very interesting properties and it turns out that they have far reaching consequences in many fields such as Economics, engineering, financial mathematics, physics. The usefulness of BFD in soliton theory is that it well describes the soliton waves and it leads to deep physical understanding. The modified extended direct algebraic method (MEDAM) is applied to obtain traveling wave solutions of Eq. (3).

The MEDAM approach is efficacious in constructing traveling wave solutions for NPDEs. This paper deals with application of the MEDAM for finding exact solutions of FBH equation and illustrating the role of the solutions in nonlinear system dynamics. The novelty of study is to investigates further development of the existing research into fractional dimensions which gained popularity particularly through the FBH equation supported by references [18-19]. The paper is organized as follows: In Section 2, we briefly discuss BFD and their properties. Section 3 gives a detailed description of the MEDAM. Section 4 discusses the implementation of the technique. In Section 5, we provide plots which illustrate some physical features of the obtained solutions. Finally, Section 6 concludes the paper.

2. Definition of BFD and properties:

In this section, we present the definition of Beta Fractional Derivative (BFD) and its properties.

Definition. The BFD is defined as [18]

Let *p(x)* be a real-valued function defined on the interval [τ,∞), the Beta fractional derivative of the function *p(x)* is defined as follows:

$$ {}_{0}^{A}D_{x}^{\beta}p(t)=\lim_{\varepsilon\to 0}\frac{p\left(x+\epsilon\left(x+\frac{1}{\Gamma(\beta)}\right)^{1-\beta}\right)-p(x)}{\epsilon},\ x>\tau,\beta\in(0,1]. $$

Theorem 3. If $0<\beta\leq 1, a, b\in\mathbb{R}$, p and q are functions β-differentiable and $\beta\in(0,1]$, the BFD satisfies the follows properties:

- ${}_{0}^{A}D_{x}^{\beta}(a\,p+bq)=a\,{}_{0}^{A}D_{x}^{\beta}(p)+\mathrm{b}\,{}_{0}^{A}D_{x}^{\beta}(q).$
- ${}_{0}^{A}D_{x}^{\beta}(a)=0.$
- ${}_{0}^{A}D_{x}^{\beta}(p\,q)=q\,{}_{0}^{A}D_{x}^{\beta}(p)+p\,{}_{0}^{A}D_{x}^{\beta}(q).$
- ${}_{0}^{A}D_{x}^{\beta}\left(\frac{p}{q}\right)=\frac{q\,{}_{0}^{A}D_{x}^{\beta}(p)-p\,{}_{0}^{A}D_{x}^{\beta}(q)}{q^{2}}.$
- ${}_{0}^{A}D_{x}^{\beta}(p\left(q(x)\right)=\left(x+\frac{1}{\Gamma(\beta)}\right)^{1-\beta}q'(x)p'(q(x)).$

The proofs of these properties are given in [18]. The BFD is a novel fractional operator that preserves some properties of classical calculus while simultaneously incorporating memory and nonlocal effects typical of fractional calculus. Owing to these properties, it is widely applied in the nonlinear modeling of physical and engineering systems. Compared with other fractional derivatives, the BFD preserves many properties of classical calculus while providing greater flexibility and accuracy in modeling memory and nonlocal behaviors in complex physical systems [18–21, 25].

3. Description of the MEDAM:

In this section, we propose the MEDAM [32-34]. This method provides a neat and elegant procedure for exact traveling-wave solutions of FBH equation. The basic concept of the method is to transform the governing nonlinear fractional partial differential equation (NFPDEs) into an ordinary differential equation (ODE) by using a suitable wave transform. Next, an algebraic ansatz is used to find explicit closed-form solutions. The main stages of the MEDAM method are briefly summarized below.

Step 1: Suppose that the FBH equation in the spatial variable and the temporal variable t is given by:

$$ Q\left(\mathrm{p},{}_{0}^{A}D_{t}^{\beta}\mathrm{p},{}_{0}^{A}D_{x}^{\beta}\mathrm{p},{}_{0}^{A}D_{tx}^{2\beta}\mathrm{p},{}_{0}^{A}D_{xx}^{\beta}\mathrm{p},{}_{0}^{A}D_{xxx}^{\beta}\mathrm{p},..\right)=0, \tag{4} $$

where $p(x,t)$ is the unknown function, and Q is a polynomial differential operator in terms of $p(x,t)$ and its corresponding fractional derivatives.

Step 2: To convert Eq. (4) into an ordinary differential equation, a suitable traveling-wave transformation is employed.

$$ p(x,t)=P(\Omega),\ \Omega=\frac{1}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}\pm\frac{\sigma}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}, \tag{5} $$

where σ a positive parameter to be determined. This transformation transforms the FBH equation into an ordinary differential equation (ODE):

$$ M(\mathrm{P},\mathrm{P}',\mathrm{P}'',\mathrm{P}''',\cdots)=0, \tag{6} $$

where M is a polynomial in $\mathrm{P}(\Omega)$, and prime (’) denotes derivative with respect to Ω.

Step 3: Let us assume that Eq. (6) has a solution in the form of

$$ \boldsymbol{P}(\Omega)=\boldsymbol{a}_{0}+\sum_{i=1}^{N}\boldsymbol{a}_{i}\Psi^{i}(\Omega)+\sum_{i=1}^{N}\boldsymbol{b}_{i}\Psi^{-i}(\Omega), \tag{7} $$

where N is a positive integer that obtained by balancing the term involving the highest-order derivative with the nonlinear term of highest power in the reduced equation (6), the coefficients a_i ($i=0,1,2,\ \ldots\ ,N$) and b_i ($i=1,2,\ \ldots\ ,N$) are constants, and $\Psi(\Omega)$ is solution of the equation

$$ \Psi'(\Omega)=\Upsilon+\Psi^{2}. \tag{8} $$

with Υ is constant.

Step 4: Eqs. (7) and (8) are inserted into Eq. (6) after the N is determined. Then the coefficients of the same powers of $\Psi(\Omega)$ are collected and set equal to zero. This process leads to an algebraic system of equations for the unknown coefficients a_0, a_i, b_i and σ that can be solved by means of symbolic computation software such as Maple. The solution sets of Eq. (8) can be divided into three cases according to the sign of Υ.

Family 1: Hyperbolic-type Solutions ($\Upsilon<0$),

$$ \Psi(\Omega)=-\sqrt{-\Upsilon}\,tanh\left(\sqrt{-\Upsilon}\Omega\right)\ \text{or}\ \Psi(\Omega)=-\sqrt{-\Upsilon}\,coth\left(\sqrt{-\Upsilon}\,\Omega\right). \tag{9} $$

Family 2: Trigonometric-type Solutions ($\Upsilon>0$),

$$ \Psi(\Omega)=\sqrt{\Upsilon}\,tan\left(\sqrt{\Upsilon}\Omega\right)\ \text{or}\ \Psi(\Omega)=-\sqrt{\Upsilon}\,cot\left(\sqrt{\Upsilon}\,\Omega\right). \tag{10} $$

Family 3: Rational-type Solutions

$$ (\Upsilon=0),\Psi(\Omega)=-\frac{1}{\Omega}. \tag{11} $$

4. Application of the method:

In this part, the MEDAM is applied to acquire the traveling wave solutions for Eq. (3)

$$ {}_0^AD_t^\beta p - {}_0^AD_x^{2\beta} p - p\, {}_0^AD_x^\beta p - p(\mu - p)(p-1) = 0. $$

Apply the following traveling wave transformation

$$ p(x,t) = P(\Omega),\ \Omega = \frac{1}{\beta}\left(x + \frac{1}{\Gamma(\beta)}\right)^\beta - \frac{\sigma}{\beta}\left(t + \frac{1}{\Gamma(\beta)}\right)^\beta. \quad (12) $$

By applying the BFD and the traveling wave transformation, we get the following equations:

${}_0^AD_t^\beta p = -\sigma P'$, ${}_0^AD_x^\beta p = P'$, ${}_0^AD_x^{2\beta} p = P''$. By inserting Eq. (12) into Eq. (3), then Eq. (3) convert to the following ODE.

$$ \sigma P' + P'' + PP' + P(\mu - P)(P-1) = 0. \quad (13) $$

By balancing the nonlinear power term P^3 with the highest order derivative P'', we get N=1. Thus, Eq. (7) is simplified as follows:

$$ P(\Omega) = a_0 + a_1\Psi(\Omega) + \frac{b_1}{\Psi(\Omega)}. \quad (14) $$

By plugging in Equation (14) into Equation (13) and grouping terms by powers of Ψ, a system of algebraic equations is found by setting the coefficient of each power of Ψ to zero. The resulting algebraic equations are solved by using the computer algebra system Maple, and the following sets of solutions are obtained.

set 1:

$$ a_0 = \frac{1}{2},\ a_1 = -1, b_1 = 0,\ \sigma = \mu - 1, \Upsilon = -\frac{1}{4}. \quad (15) $$

set 2:

$$ a_0 = \frac{\mu}{2} + \frac{1}{2},\ a_1 = 2, b_1 = 0,\ \sigma = \frac{\mu}{2} + \frac{1}{2} $$
$$ \Upsilon = -\frac{\mu^2}{16} + \frac{1}{8}\mu - \frac{1}{16} = -\frac{(\mu-1)^2}{16}, (\mu - 1 \neq 0). \quad (16) $$

set 3:

$$ a_0 = \frac{\mu}{2} + \frac{1}{2},\ a_1 = -1, b_1 = 0,\ \sigma = -\mu - 1, $$
$$ \Upsilon = -\frac{\mu^2}{4} + \frac{1}{2}\mu - \frac{1}{4} = -\frac{(\mu-1)^2}{4}, (\mu - 1 \neq 0). \quad (17) $$

set 4:

$$ a_0 = \frac{\mu}{2},\ a_1 = 2, b_1 = 0,\ \sigma = \frac{\mu}{2} - 2, \Upsilon = -\frac{\mu^2}{16}. \quad (18) $$

set 5:

$$ a_0 = \frac{1}{2},\ a_1 = 2, b_1 = 0,\ \sigma = \frac{1}{2} - 2\mu, \Upsilon = -\frac{1}{16}. \quad (19) $$

set 6:

$$ a_0 = \frac{\mu}{2},\ a_1 = -1, b_1 = 0,\ \sigma = -\mu + 1, \Upsilon = -\frac{\mu^2}{4}. \quad (20) $$

The wave Solutions expressions for *P(x,t)* are given below set by set

Set 1: For the solution families presented below, the variable Ω is defined as

$$ \Omega = \frac{1}{\beta}\left(x + \frac{1}{\Gamma(\beta)}\right)^\beta + \frac{\mu-1}{\beta}\left(t + \frac{1}{\Gamma(\beta)}\right)^\beta, \text{ and } \Upsilon = -\frac{1}{4} $$

Family 1: Hyperbolic-type Solutions Υ<0,

$$ P_1(\Omega) = \frac{1}{2} + \sqrt{-\Upsilon}\, tanh(\sqrt{-\Upsilon}\Omega) \text{ or } P_2(\Omega) = \frac{1}{2} + \sqrt{-\Upsilon}\, coth(\sqrt{-\Upsilon}\Omega). $$

Family 2: Trigonometric-type Solutions Υ >0,

$$ P_3(\Omega) = \frac{1}{2} + \sqrt{-\Upsilon}\, tan(\sqrt{-\Upsilon}\Omega) \text{ or } P_4(\Omega) = \frac{1}{2} + \sqrt{-\Upsilon}\, cot(\sqrt{-\Upsilon}\Omega). $$

Family 3: Rational-type Solutions Υ = 0,

$$ P_5(\Omega) = \frac{1}{2} - \frac{1}{\Omega}. $$

Set 2: The solution families presented below are expressed in terms of the traveling-wave variable Ω defined by

$$ \Omega = \frac{1}{\beta}\left(x + \frac{1}{\Gamma(\beta)}\right)^\beta - \frac{\left(\frac{\mu}{2}+\frac{1}{2}\right)}{\beta}\left(t + \frac{1}{\Gamma(\beta)}\right)^\beta \text{with } \Upsilon = -\frac{(\mu-1)^2}{16} $$

Family 1: Hyperbolic-type Solutions Υ<0

$$ P_6(\Omega) = \frac{\mu}{2} + \frac{1}{2} - 2\sqrt{-\Upsilon}\, tanh(\sqrt{-\Upsilon}\Omega) \text{ or } P_7(\Omega) = \frac{\mu}{2} + \frac{1}{2} - 2\sqrt{-\Upsilon}\, coth(\sqrt{-\Upsilon}\,\Omega). $$

Family 2: Trigonometric-type Solutions Υ >0,

$$ P_8(\Omega) = \frac{\mu}{2} + \frac{1}{2} + 2\sqrt{\Upsilon}\, tan(\sqrt{\Upsilon}\Omega) \text{ or } P_9(\Omega) = \frac{\mu}{2} + \frac{1}{2} - 2\sqrt{\Upsilon}\, coth(\sqrt{\Upsilon}\,\Omega). $$

Family 3: Rational-type Solutions ϒ=0,

$$P_{10}(\Omega) = \frac{\mu}{2} + \frac{1}{2} + \frac{2}{\Omega}.$$

Set 3: For all solution families listed below

$$\Omega = \frac{1}{\beta}\left(x + \frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{(\mu+1)}{\beta}\left(t + \frac{1}{\Gamma(\beta)}\right)^{\beta}$$

and $\Upsilon = -\frac{(\mu-1)^2}{4}$. The obtained solutions are written in terms of the variable Ω and the parameter ϒ.

Family 1: Hyperbolic-type Solutions ϒ<0,

$$P_{11}(\Omega) = \frac{\mu}{2} + \frac{1}{2} + \sqrt{-\Upsilon}\, tanh\left(\sqrt{-\Upsilon}\Omega\right).$$

Or $P_{12} = \frac{\mu}{2} + \frac{1}{2} + \sqrt{-\Upsilon}\, coth\left(\sqrt{-\Upsilon}\,\Omega\right)$.

Family 2: Trigonometric-type Solutions ϒ>0,

$$P_{13}(\Omega) = \frac{\mu}{2} + \frac{1}{2} - \sqrt{\Upsilon}\, tan\left(\sqrt{\Upsilon}\Omega\right).$$

Or $P_{14}(\Omega) = \frac{\mu}{2} + \frac{1}{2} + \sqrt{\Upsilon}\, cot\left(\sqrt{\Upsilon}\,\Omega\right)$.

$$P_{15}(\Omega) = \frac{\mu}{2} + \frac{1}{2} - \frac{1}{\Omega}.$$

Utilizing the approach used for the preceding sets, we can similarly find wave solutions for sets (4-6). The computational details are omitted for brevity and to avoid repetition.

5. Graphical representations of the solutions:

Graphical visualizations of the obtained wave solutions are generated for solutions $P_1(x,t), P_8(x,t)$ and $P_{15}(x,t)$ using Maple software to demonstrate their dynamic behaviors and structural characteristics. The profiles of the waves show that the propagation mode and the amplitude can be modulated by the free parameters of the solutions, and thus the corresponding surfaces in the graphical spaces are significantly changed. To study the effect of the fractional order, we set

$$\mu = 2, \text{ with } x \in [0,10] \text{ and } t \in [0,10].$$

Figure 1: Panels (a)-(c) depict 3D surface plots and panel (d) shows the corresponding 2D profile of the wave solution for , and , respectively. The 3D plots reveal that when the fractional order β decays from 1 to 0.90, the amplitude of traveling wave 3D diminishes, the profile becomes more unlocalized, and the effective wave speed becomes slower. The 2D graph also demonstrates that smaller values of β lead to the close spacing among the curves indicates that the influence of β is moderate. The smooth and bounded nature of the curves confirms the stability of the solution over the considered time interval.

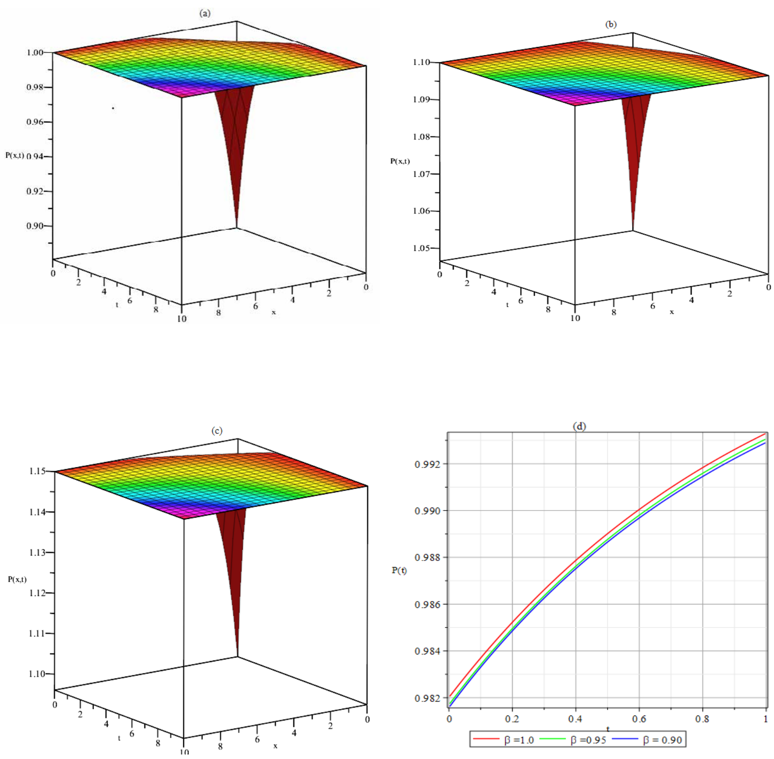

Figure 1: ***3D plots (a)–(c) of the solution*** P1 (x,t) ***for μ=2, and β=1,0.95,0.90, respectively, followed by the corresponding 2D plot at x=2.***

Figure 2: panels (a)–(c) depict 3D surface plots and panel (d) shows the corresponding 2D profile of the wave solution $P_8\,(x,t)$ for β =1,0.95, and 0.90 respectively. In the 3D plots, the solution exhibits an increase in the peak amplitude and the formation of steep fronts as the fractional order decreases from β =1 to β =0.90 and ϒ=1, the wave velocity is observed to vary with β\betaβ, consistent with the β -dependent traveling-wave coordinate. In panel (d), the amplitude decays with time. Moreover, as β decreases from 1 to 0.90, the 2D waveform becomes more localized and steeper.

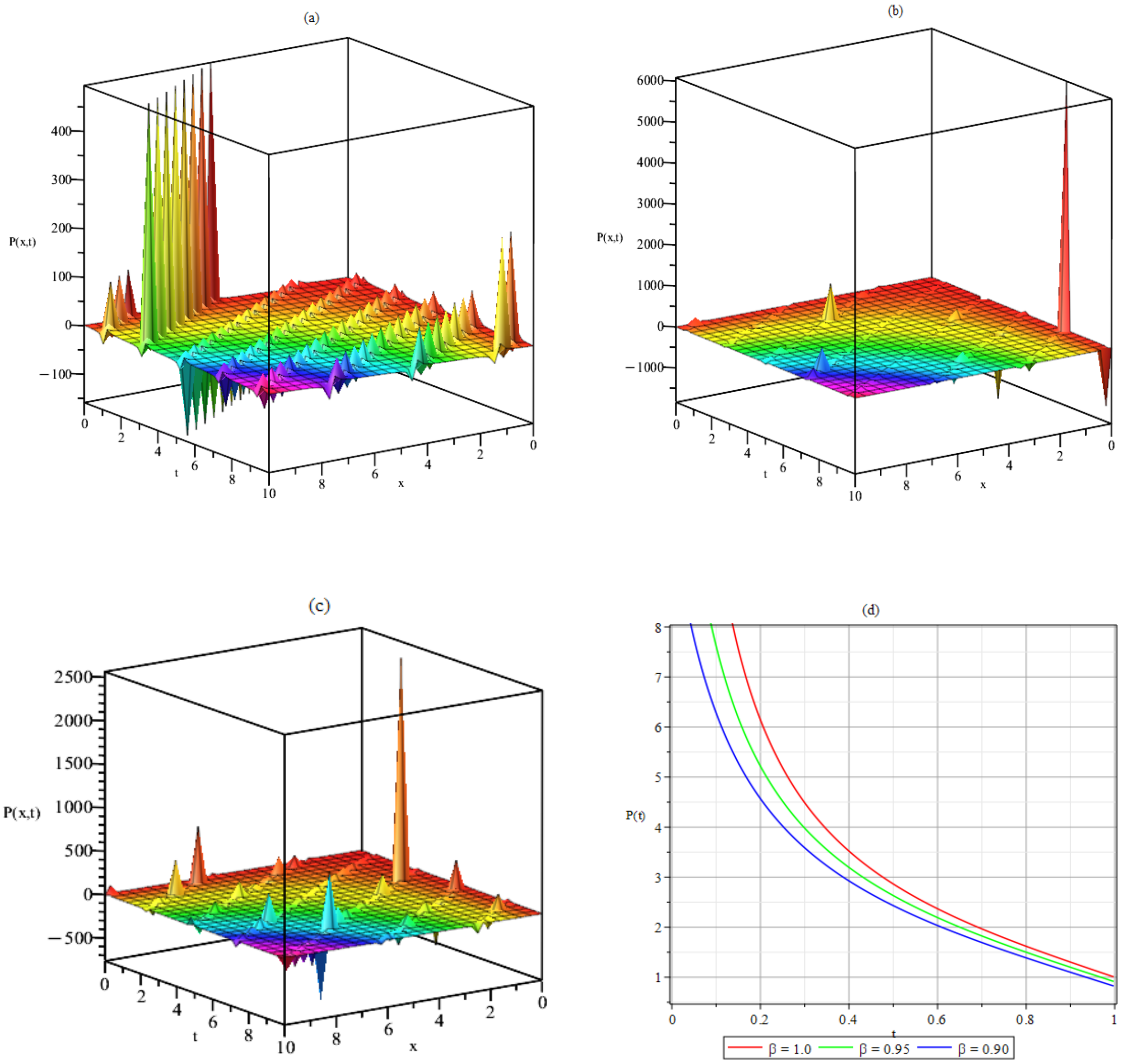

Figure 2: 3D plots (a)–(c) of the solution P_8 *(x, t) for β =1,0.95*,0.90, respectively, followed by the corresponding 2D plot at x=2.

Figure 3: Shows the solution $P_{15}\,(x,t)$. Panels (a)- (c) show 3D plots of the wave solution for fractional order β=1, 0.95 and 0.90, respectively, and the corresponding 2D profiles are displayed in (d). In panels (a)-(c), the amplitude is gradually decreased and the wave surface gets smoother with vanished spatial oscillations when the fractional order drops from β =1 to β=0.90. This demonstrates that the memory effect enhancement inhibits the wave propagation and the nonlinear fractional medium becomes more dissipative. The 2D profiles further verify that smaller fractional-order values result in a more rapid decline of the wave amplitude with time.

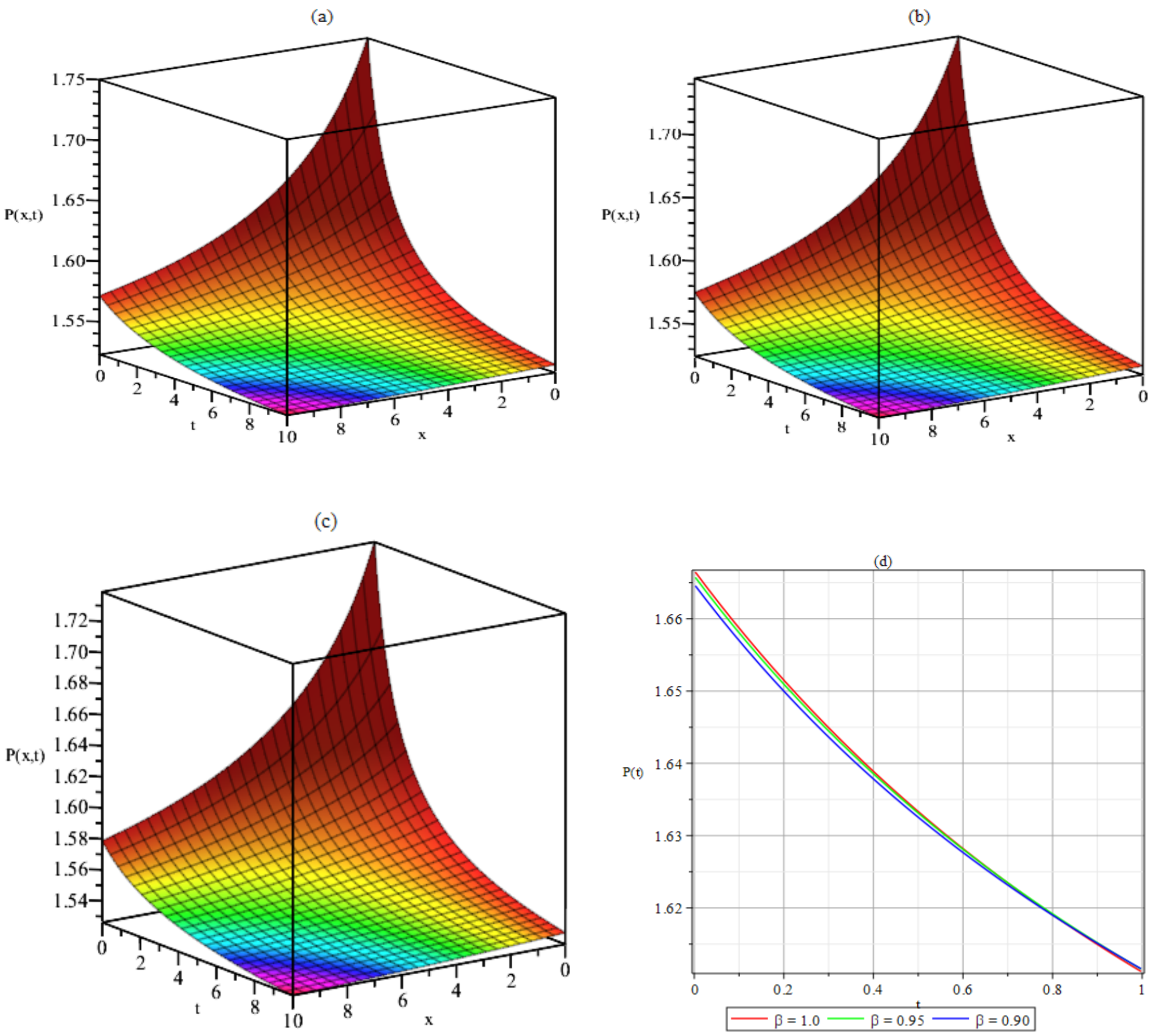

Figure 3: ***3D plots (a)–(c) of the solution $P_{15}(x,t)$ for $\mu=2$ and $\beta=1,0.95,0.90$, respectively, followed by the corresponding 2D plot at $x=2$.***

6. Discussion and conclusion

In this study, exact traveling-wave solutions of the FBH equation were derived using the MEDAM within the BFD framework. Hyperbolic, trigonometric, and rational solution families were obtained, revealing rich nonlinear wave dynamics. The graphical analysis demonstrated that the fractional-order parameter significantly affects the wave amplitude, localization, dissipation, and propagation behavior, where lower fractional-order values produce stronger memory effects and smoother localized wave profiles.

The obtained results are consistent with previous studies on nonlinear fractional differential equations and wave propagation models [10, 14, 15, 32–36]. Similar influences of the fractional-order parameter on wave attenuation and propagation behavior have been reported in earlier investigations of fractional Burgers-type and nonlinear evolution equations. In particular, Bilal et al. [32], Ghayad et al. [33], and Hubert et al. [34] confirmed the effectiveness of the MEDAM in constructing explicit analytical wave and soliton solutions without relying on perturbation or iterative procedures. Compared with other analytical approaches, such as the Kudryashov method, the Riccati–Bernoulli sub-ODE method, the modified extended tanh-function method, the extended expansion method, and the homotopy perturbation method [10, 13, 14, 16, 17], the MEDAM provides a more direct, systematic, and efficient framework for obtaining exact traveling-wave solutions. Furthermore, the incorporation of the BFD improves the modeling of memory and nonlocal effects in nonlinear media.

From a physical viewpoint, the obtained solutions provide valuable insight into nonlinear transport and reaction-diffusion phenomena governed by convection, diffusion, and reaction mechanisms. Therefore, the FBH model may contribute to a better understanding of nonlinear processes arising in fluid dynamics, plasma physics, biological wave propagation, and chemical reaction systems. Moreover, the Beta fractional derivative offers greater flexibility in modeling memory and hereditary effects that are difficult to capture using classical integer-order differential models.

One of the major strengths of the present work is the use of the BFD, which preserves important properties of classical calculus while simultaneously incorporating fractional memory and nonlocal behaviors. In addition, the MEDAM proved to be an efficient and reliable technique for deriving multiple exact traveling-wave solutions and for analyzing the influence of the fractional-order parameter on nonlinear wave dynamics. Moreover, the theoretical questions concerning the existence, uniqueness, stability, and well-posedness of the solutions of the FBH equation remain open and require further mathematical investigation. Although the present MEDAM results provide explicit closed-form wave structures that compare favorably with other analytical and numerical approaches available in the literature, a rigorous well-posedness and stability analysis would further strengthen the mathematical validity and physical reliability of the obtained solutions [18, 25, 35, 36].

Future research may extend the present analysis to higher-dimensional, variable-coefficient, and stochastic FBH models, together with numerical simulations and stability investigations of the derived solutions. Comparative studies involving other analytical and computational techniques may also provide additional insight into the applicability and robustness of the MEDAM. Overall, the present results confirm that the MEDAM is an effective and reliable method for constructing exact solutions of nonlinear fractional differential equations, while emphasizing the important role of fractional-order effects in modeling nonlinear systems with memory and nonlocal characteristics.

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