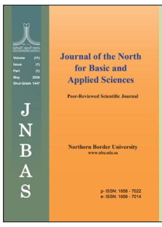

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Analysis of Critical Buckling Moments in Steel I-Girders with Initial Imperfections

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Abstract

The critical buckling moment is a fundamental parameter for the analysis of steel I-girders, particularly when accounting for initial imperfections that can significantly influence structural performance. This study presents a novel approach to accurately compute the critical buckling moments of steel I-girders by integrating the Spencer technique with finite element analysis (FEA) using ANSYS 20. This combined methodology leverages detailed deflection-load data from the FE model to apply the Spencer technique, which is traditionally used for plates, to the more complex I-girder geometry. A comprehensive parametric study was conducted to examine the influence of web slenderness (ranging from h_w/t_w= 142 to 200) and flange compactness (covering both compact and non-compact scenarios) at the critical moment. The results demonstrate that the proposed integrated approach provides a robust and precise estimation of the critical buckling moment, showing good agreement with established theoretical and numerical methods, thereby validating its application. Crucially, the findings provide specific insights into the required safety margins for I-girders with initial imperfections, offering engineers a practical tool for complying with design codes such as SBC 306 and ECP 205.

Keywords: Bending Stresses; Buckling Behavior; Theoretical Analysis; Steel I-girders; Plate Girders.

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Nomenclature

b_f	Flange width
c	The half-width of the flange
t_f	Flange thickness
d	Overall depth of girder
h_w	Web height
t_w	Web thickness of plate girder
w_o	Initial imperfection functions.
w	Deflections functions of plate.
H1	The first transformed variable, representing the deflection ratio
H2	The second transformed variable, representing the load-deflection ratio.
F1	A correction factor related to the total deflection
F2	A correction factor related to the initial imperfection

1. Introduction

Steel I-girders are fundamental components in structural engineering, valued for their high strength-to-weight ratio and efficiency in resisting bending moment and shear forces. However, their stability under load is a critical design consideration. Buckling, characterized by sudden lateral deflection or twisting, represents a significant failure mode, particularly under bending moments. The accurate determination of the critical buckling moment is therefore essential for ensuring the structural integrity and safety of these elements.

1.1 Initial Geometric Imperfections of Steel Plate Girders

The manufacturing and fabrication processes inherently introduce initial geometric imperfections in steel plate girders. These imperfections can significantly reduce the buckling capacity by leading to localized stress concentrations and premature failure [1]. Consequently, the accurate modeling and consideration of these initial defects are paramount in modern structural analysis.

Over the years, numerous studies have focused on quantifying and incorporating these imperfections into both experimental and numerical investigations. The rise of sophisticated finite element analysis (FEA) programs, such as ANSYS, has transformed the process of imperfections into a complex, strategic task [2-3]. However, current design guidelines, such as EN 1993-1-5 [2-4], recommend that the chosen imperfection should correspond to the mode of least resistance, often employing buckling mode-affine or hand-defined sinusoidal geometric flaws [5-7]. Research has explored the use of various half-wavelengths to define local geometric imperfection shapes [8]. For instance, Boissonnade and Somja [9] showed that for a variety of cross-sections, setting the maximum imperfection amplitude to 0.1 times the plate thickness or the plate width divided by 200 resulted in negligible differences in local ultimate capacities. Eurocode 3 Part 1-5, which addresses plated structures [10], advises using an eigenmode shape obtained from an initial linear buckling analysis, scaled by a factor that correlates with the lowest ultimate strength [11-12].

More recent research continues to explore the complex interaction between imperfections and buckling behavior. Studies have investigated the influence of initial imperfections and residual stress on the lateral-torsional buckling of steel I-beams [13], while others have focused on the buckling strength of I-beams under extreme conditions, such as elevated temperatures [14]. Furthermore, the effect of initial geometric imperfections on the ultimate and post-buckling shear strengths of I-girders with web discontinuities has also been a subject of recent investigation [15]. The experimental and numerical analysis of high-strength stainless steel I-girders further highlights the ongoing need for accurate imperfection modeling in advanced materials [16].

The analysis of critical buckling moments in steel I-girders with initial defects is the primary goal of this study. The study examines how the size, shape, and distribution of imperfections affect critical buckling performance using both analytical techniques and numerical simulations.

1.2 Using the Southwell and Spencer Techniques to Calculate the Buckling Load.

The Southwell method [17] was developed to predict the buckling load of initially flawed columns without requiring failure testing. It relies on a specific linear relationship between the deflection-to-load ratio and the deflection itself, derived from the load-deflection data [18-20]. However, experiments have shown that the Southwell approach can occasionally produce non-linearities, particularly at low and high load categories [21].

To address these limitations, Spencer and Walker [21] proposed an alternative technique, known as the Spencer technique, which uses "pivot points" to linearize the data and provide a more robust estimation of the critical load for plates with minor flaws. This technique has proven to be a valuable tool for extracting the elastic critical load from experimental or numerical load-deflection data, especially in the presence of imperfections.

The current study aims to compute the critical buckling moments for steel I-girders by integrating the Spencer technique with deflection-load data obtained from finite element program analysis (ANSYS). This combined approach allows for a precise estimation of the elastic buckling load for plate girders under pure moment, considering the effect of initial imperfections and boundary conditions. A comprehensive parametric analysis is conducted, considering both material properties and geometric characteristics of the girders. Finally, the precision of applying these techniques is verified by judgment with several methods. The novelty of this work lies in the systematic application of the Spencer technique to a wide range of FE models of I-girders with varying geometric imperfections, providing a refined methodology for stability analysis relevant to modern design codes like the Saudi Building Code (SBC 306 CR) [22] and the Egyptian Code (ECP 205) [23].

While previous studies have focused on either finite element modeling of I-girder buckling or the theoretical application of techniques such as Southwell's to simple plates, a significant gap remains in the integrated, practical application of advanced theoretical methods to complex FEA results for I-girders with imperfections. Specifically, the Spencer technique, which offers a robust method for estimating critical loads from load-deflection data, has not been systematically applied to the numerical output of full-scale I-girder models. The primary novelty of this research is the pioneering integration of the Spencer technique with finite element simulation (ANSYS 20) to accurately determine the critical buckling moments of steel I-girders with initial imperfections. This combined approach not only enhances the precision of critical moment prediction but also provides a novel, validated methodology that can be adopted by researchers and practitioners to bridge the gap between theoretical stability analysis and complex numerical modeling in structural engineering.

2. Research Strategy and Methodology

The finite element analysis was performed using ANSYS Mechanical APDL Academic Research Version 20.0, for non-commercial academic research purposes. This combined approach significantly improves prediction accuracy while offering valuable insights into the stability and performance of structural components under diverse loading conditions. The detailed deflection analysis provided by ANSYS 20 highlights key patterns essential for understanding material buckling behavior, and the refinement offered by the Spencer technique ensures that the results are both precise and applicable in practical engineering scenarios.

2.1 Finite Element Modeling

ANSYS 20 was used to create precise finite element models that included all relevant characteristics, including nodes, elements, material properties, dimensions, and boundary conditions, to theoretically analyze how thin plate girders would behave under bending force.

To ensure accuracy in replicating the actual structural dimensions, the modeling procedure began by defining the plate girder's geometric parameters, such as flange width, web height, web thickness, and overall girder length. Young's modulus, Poisson's ratio, and yield strength were among the material attributes carefully selected to represent the steel's physical characteristics.

Boundary conditions were applied to simulate realistic support and loading scenarios, including lateral constraints and simply supported ends. The meshing of the model was carried out at a high density to ensure precise node and element distributions, capturing intricate stress and deflection patterns. This systematic approach ensures that the ANSYS models accurately replicate the physical behavior of slender plate girders under bending, providing a robust foundation for analyzing their critical buckling behavior.

2.1.1 Building models and analysis

For the 3D modeling of the I-girder, the SOLID185 Structural Solid element was utilized. This element, illustrated in Figure (1), is an eight-node element with three degrees of freedom per node, capable of handling complex behaviors such as plasticity, large deflection, and large strain analysis, which are essential for accurately simulating the buckling phenomenon.

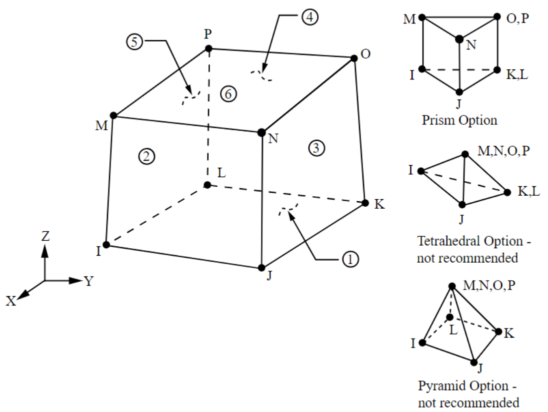

Figure 1: Homogeneous Structural Solid Geometry SOLID185

2.1.2 Verification of the Finite Element Model:

The reliability of the FE model was established through a two-step process: (1) Model Verification (Mesh Sensitivity) and (2) Model Validation (Comparison with Theoretical Solution). Model Verification (Mesh Sensitivity Analysis): The accuracy of the finite element analysis was first confirmed through a mesh sensitivity study. This involved testing a sample plate girder model under the same loading conditions using both a coarse mesh and a fine mesh (with a 30% increase in density). The comparison showed that the difference in the critical output parameter (peak stress) between the two meshes was negligible, at only 0.7%. To optimize computational efficiency without sacrificing accuracy, the coarser mesh was selected for the subsequent parametric study. Model Validation (Comparison with Theoretical Solution): The model was validated by comparing its results against a well-established theoretical solution for a simply supported beam under point loads. Specifically, the maximum deflection (δ) obtained from the FE model was compared to the theoretical deflection calculated using the standard beam theory formula:

$$\delta = \frac{PL^3}{48EI} + \frac{kPL}{4GA}$$

As shown in Figure (2), the maximum deflection obtained from the FE model was 0.001891, which resulted in a minimal percentage error of 1.6% when compared to the theoretical solution (δ=0.00186cm). This close agreement confirms the accuracy and reliability of the chosen element type, mesh density, and boundary conditions for the subsequent parametric study.

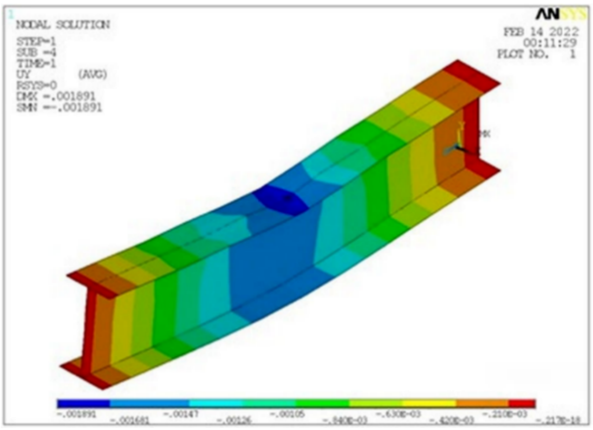

Figure (2) Displacement at Y direction for vitrificated model

2.2 Parameters of the analytical model

This section of the study presents a numerical investigation using finite element analysis, applying the Spencer technique to simulated data. The primary variable examined was the depth-to-web thickness ratio of girders, ranging from 200 to 142, with three ratios representing slender webs and five ratios representing the outstanding length of the flange for plate girders, varying from 5.8 to 9.8 to cover both compact and non-compact flange scenarios. All these cases were evaluated in accordance with the Saudi Building Code (SBC306) [24-25] and the Egyptian Code (ECP205) [25]. Additionally, this study included variations in the aspect ratio of plate girder panels, considering aspect ratios of 1 and 0.6, with initial imperfections introduced as percentages of web thickness (20% and 40%). The study ultimately highlights the most influential cases, which will be discussed in subsequent sections.

The following parameters of the plate girder's cross-section variation were considered in the theoretical analysis and were summarized in Tables 1 and 2:

Table (1) Plate girder model dimensions for cases of compact flanges

Model	h_w	t_w	b_f	t_f	h_w/t_w=	c/t_f
ACMF1	100	0.5	24	2	200	5.875
ACMF2	100	0.5	24	1.8	200	6.52778
ACMF3	100	0.5	24	1.6	200	7.34375
BCMF1	100	0.6	24	2	166	5.85
BCMF2	100	0.6	24	1.8	166	6.5
BCMF3	100	0.6	24	1.6	166	7.3125
CCMF1	100	0.7	24	2	142	5.825
CCMF2	100	0.7	24	1.8	142	6.47222
CCMF3	100	0.7	24	1.6	142	7.28125

*All cases are studied for aspect ratio =0.6 and initial imperfection of web =40% of the web thickness

Table (2) Plate girder model dimensions for cases of non-compact flanges

Model	h_w	t_w	b_f	t_f	h_w/t_w=	c/t_f
ANC1	100	0.5	24	1.4	200	8.39286
ANCF2	100	0.5	24	1.2	200	9.79167
BNCF1	100	0.6	24	1.4	166	8.35714
BNCF2	100	0.6	24	1.2	166	9.75
CNCF1	100	0.7	24	1.4	142	8.32143
CNCF2	100	0.7	24	1.2	142	9.70833

*All cases are studied for aspect ratio =0.6 and initial imperfection of web =40% of the web thickness

The analytical model used for the FE simulations is a simply supported symmetric girder subjected to two-point loading at the third locations, which creates a pure bending moment region in the central span. Figure (3) provides a schematic representation of this setup, clearly defining the basic geometric parameters (b_f, t_f, h_w, t_w, etc.) used in the analysis and summarized in Tables (1) and (2).

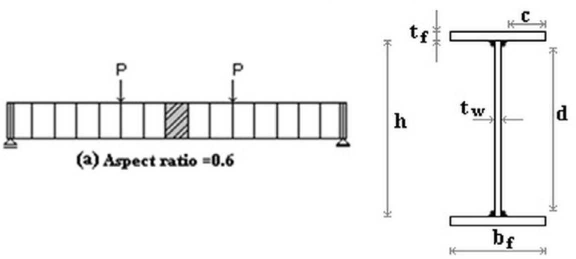

Figure (3) Basic Parameters of the analytical model

3. Analysis of theoretical results and discussions

Evaluating critical buckling moments for plate girders under pure bending is a crucial aspect of structural stability analysis, and the Spencer technique provides an effective framework for this purpose. This method integrates theoretical principles with computational precision to evaluate the moment at which the girder transitions from stability to instability. The critical buckling behavior of thin plate girders was examined in this study using the Spencer technique, which considered the geometry, material properties, and boundary conditions of these girders.

3.1 Evaluating critical buckling moment for slender I-girder with compact flange.

Section 3.1 presents the application of the Spencer technique to the FEA results for slender I-girders with compact flanges. Figures (4-a) through (6-c) illustrate the linear relationship between H1 and H2 for various geometric cases (e.g., =200, =167 and =143). The critical buckling load () for each case is derived directly from the slope of the best linear fit shown in these figures, which is a key step in determining the critical buckling moment.

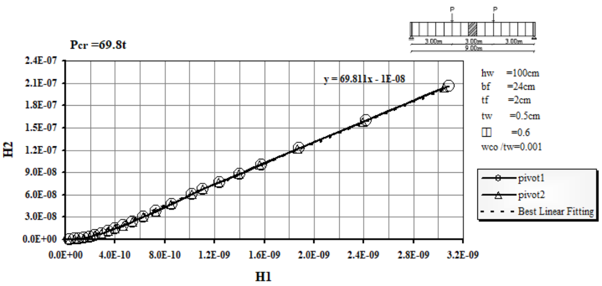

Figure (4-a) H1-H2 Relationship for the evaluation of critical moment from Spencer technique (ACMF1, =200, ϕ=0.6)

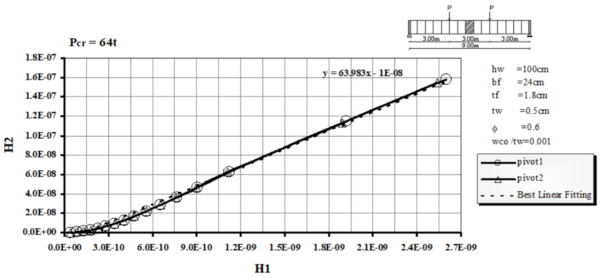

Figure (4-b) H1-H2 Relationship for the evaluation of critical moment from Spencer technique (ACMF2, =200, ϕ=0.6)

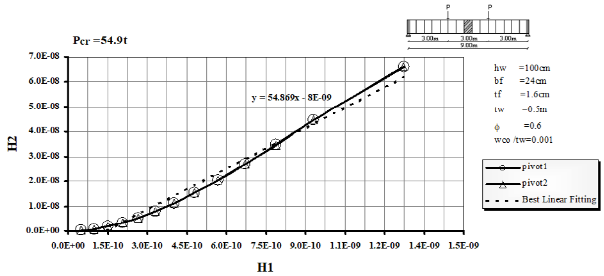

Figure (4-c) H1-H2 Relationship for the evaluation of critical moment from Spencer technique (ACMF3, =200, ϕ=0.6)

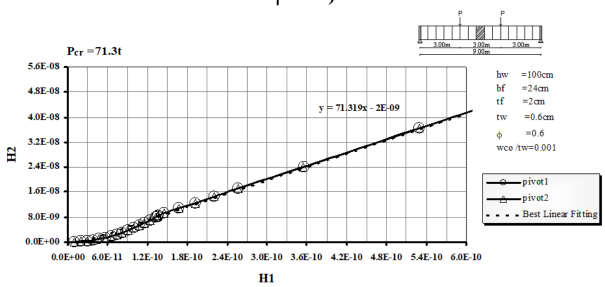

Figure (5-a) H1-H2 Relationship for the determination of critical load from Spencer technique (BCMF1, =167, ϕ=0.6)

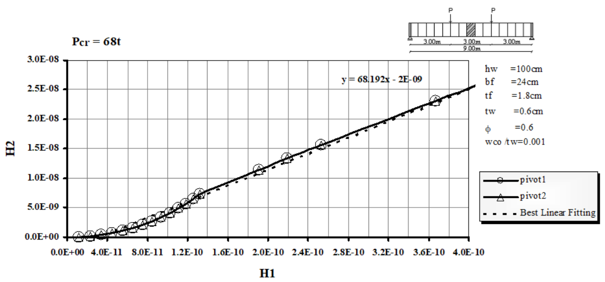

Figure (5-b) H1-H2 Relationship for the determination of critical load from Spencer technique (BCMF2, =167, ϕ=0.6)

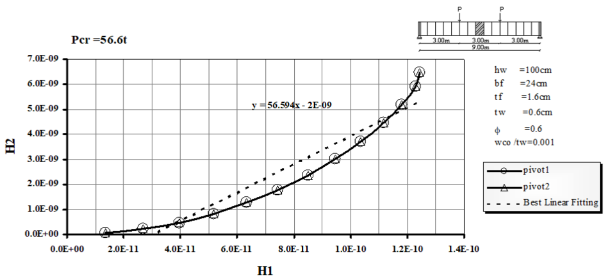

Figure (5-c) H1-H2 Relationship for the determination of critical load from Spencer technique (BCMF3, =167, ϕ=0.6)

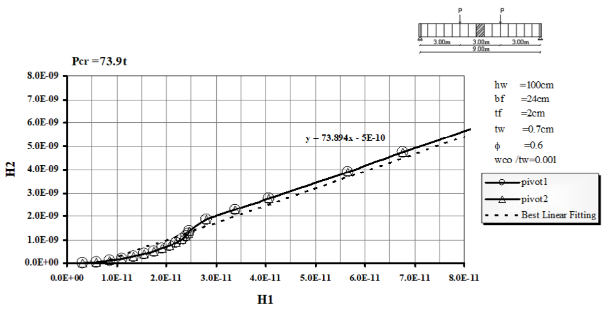

Figure (6-a) H1-H2 Relationship for the determination of critical load from Spencer technique (CCMF1, =143, ϕ=0.6)

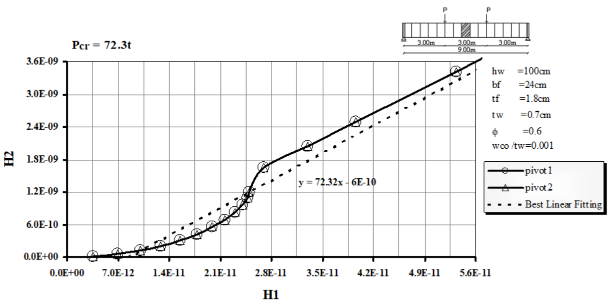

Figure (6-b) H1-H2 Relationship for the determination of critical load from Spencer technique (CCMF2, =143, ϕ=0.6)

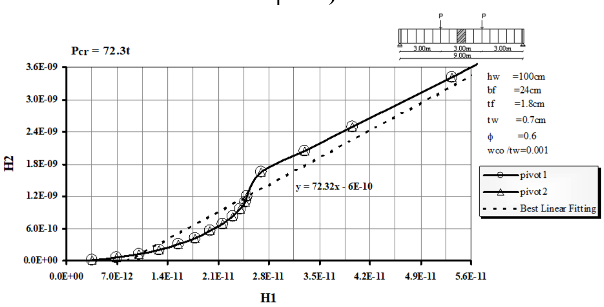

Figure (6-c) H1-H2 Relationship for the determination of critical load from Spencer technique (CCMF3, =143, ϕ=0.6)

From previous figures, it was noticed that all cases of compact flange were aligned with the best linear fitting from the Excel program at Figs (4-a), (4-b), (4-c), and Figs (5-a), (5-b) except Fig. (5-c), that when applying Spencer technique, the relation between H1 and H2 were not linear as the compactness of flange approaches to the case of the non-compact flange. Also, it was noticed in Fig. (6-b) and Fig. (6-c), that when applying the Spencer technique, the relation between H1 and H2 was not linear as the compactness of the flange approaches the case of the non-compact flange. The critical load produced from applying this technique is summarized in Table 3) as follows.

Table (3) Critical load of Plate girder models with compact flanges using the Spencer technique

Model	h_w	t_w	b_f	t_f	$h_w/t_w=$	c/t_f	*Pcr*
ACM1	100	0.5	24	2	200	5.875	69.8
ACM2	100	0.5	24	1.8	200	6.52778	64
ACM3	100	0.5	24	1.6	200	7.34375	54.9
BCM1	100	0.6	24	2	166	5.85	71.3
BCM2	100	0.6	24	1.8	166	6.5	68
BCM3	100	0.6	24	1.6	166	7.3125	56.6
CCM1	100	0.7	24	2	142	5.825	73.9
CCM2	100	0.7	24	1.8	142	6.47222	72.3
CCM3	100	0.7	24	1.6	142	7.28125	67

The observed reduction in the critical buckling moment with increasing web slenderness (h_w/t_w)) is a direct consequence of the reduced flexural and shear stiffness of the web plate. According to classical plate buckling theory, the critical stress of a plate is inversely proportional to the square of its slenderness. As h_w/t_w) increases, the web becomes more susceptible to local web buckling under the compressive stresses induced by the bending moment. The successful application of the Spencer technique, as evidenced by the clear linear relationship in the H1-H2 plots (Figures 6-8), validates its utility for extracting the elastic critical load from non-linear FEA data. The technique's ability to use 'pivot points' effectively filters out the non-linear effects caused by the initial imperfections and the onset of plasticity, which typically plague the traditional Southwell plot at high and low loads. By isolating elastic behavior, the Spencer method provides a robust and reliable estimate of the theoretical elastic critical moment, , which is a crucial input for design code formulas that account for inelastic and post-buckling reserve strength

3.2 Evaluating critical buckling moment for slender I-girder with non-compact flange

In this section, Figs (7-a), (7-b), Figs (8-a), (8-b), and Figs (9-a), (9-b) represent relationships between H1 and H2 for the two cases of non-compact flange with =200, =167 and =143, respectively. All cases with an aspect ratio (ϕ=0.6) and an initial imperfection were 40% of the web thickness.

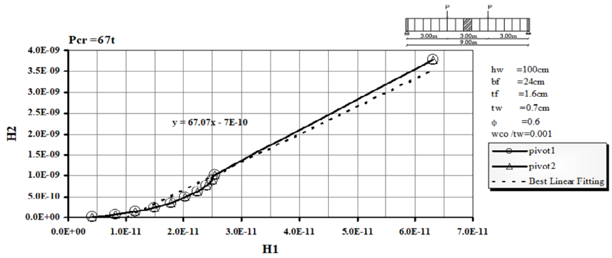

Figure (7-a) H1-H2 Relationship for the determination of critical load from Spencer technique (NCF1, =200, ϕ=0.6)

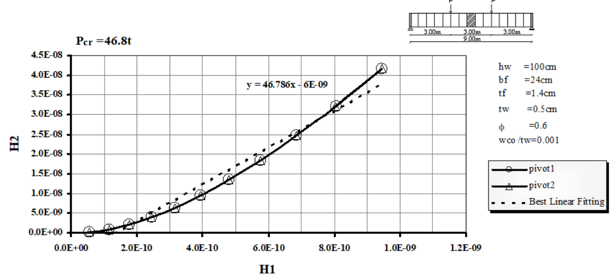

Figure (7-b) H1-H2 Relationship for the determination of critical load from Spencer technique (NCF2, =200, ϕ=0.6)

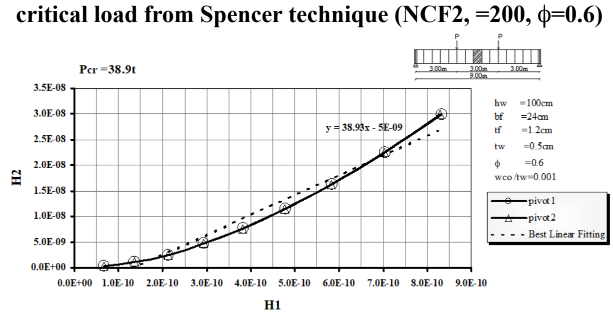

Figure (8-a) H1-H2 Relationship for the determination of critical load from Spencer technique (BNCF1, =167, ϕ=0.6)

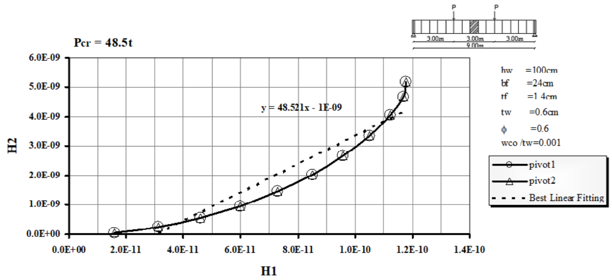

Figure (8-b) H1-H2 Relationship for the determination of critical load from Spencer technique (BNCF2, =167, ϕ=0.6)

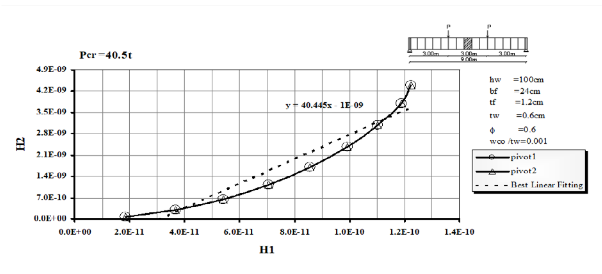

Figure (9-a) H1-H2 Relationship for the determination of critical load from Spencer technique (CNC1, =143, ϕ=0.6)

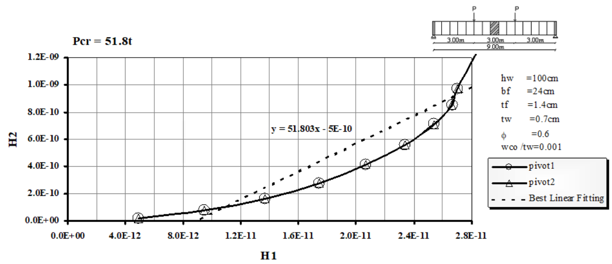

Figure (9-b) H1-H2 Relationship for the determination of critical load from Spencer technique (CNC2, =143, ϕ=0.6)

From previous figures, it was noticed that all cases of non-compact flange were not correctly aligned with the best linear fitting from the Excel program at Figs (7-a), Figs (7-b), Figs (8-a), (8-b), and Figs (9-a), (9-b). It was noticed that when applying the Spencer technique, the relation between H1 and H2 was not linear, as in the case of the compact flange. This resulted from the constraint of the flange to the web transmitted from the case of fixed to partially fixed [26-27]. The critical load produced from applying this technique is summarized in Table 4 as follows: -

Table (4) Critical load of Plate girder models with non-compact flanges using the Spencer technique

Model	h_w	t_w	b_f	t_f	$h_w/t_w=$	c/t_f	*Pcr*
ANC1	100	0.5	24	1.4	200	8.39286	46.8
ANC2	100	0.5	24	1.2	200	9.79167	38.9
BNC1	100	0.6	24	1.4	166	8.35714	48.5
BNC2	100	0.6	24	1.2	166	9.75	40.5
CNC1	100	0.7	24	1.4	142	8.32143	51.8
CNC2	100	0.7	24	1.2	142	9.70833	52.3

The flange slenderness ratio (c/) is a critical factor because the compression flange provides the primary lateral stiffness required to resist LTB. The critical moment is susceptible to the flange's ability to remain stable. When the flange slenderness increases (i.e., the flange becomes less compact), it becomes prone to local flange buckling before the overall LTB of the girder can be fully mobilized. This premature local buckling effectively reduces the cross-sectional area and the moment of inertia of the compression flange, leading to a reduction in the effective lateral stiffness of the girder. Therefore, the girder reaches its critical buckling state at a lower applied moment, confirming the necessity of design code limits (such as those in SBC 306 and ECP 205) on flange slenderness to ensure the full LTB capacity is achieved.

3.3 Comparison results of the present work with the load square deflection method

According to Venkataramaiah and Roorda [28], the load vs. lateral displacement squared approach can be used to calculate the elastic critical buckling stress. The elastic buckling load in the case of pure compression is represented by the intersection of the tangent with the load axis, which is drawn to the curve in the post-buckling area in this method. Furthermore, linear curve fitting in the post-buckling range yields the experimental post-buckling slope of the load versus maximum squared web deflection (P versus w^2), which sheds light on buckling behavior and structural stability [14-15, 29-30]. Table 5 and Table 6 show a comparison between the present method and the load square deflection method: -

Table (5) Comparison results of the present work with the load square deflection method for a plate girder with a compact flange

Model	h_w/t_w=	c/t_f	Pcr (P.W.)	Pcr (L.S.D.M)
ACM1	200	5.875	69.8	69
ACM2	200	6.52778	64	63
ACM3	200	7.34375	54.9	57
BCM1	166	5.85	71.3	72
BCM2	166	6.5	68	66
BCM3	166	7.3125	56.6	60
CCM1	142	5.825	73.9	75
CCM2	142	6.47222	72.3	69
CCM3	142	7.28125	67	63

Table (6) Comparison results of present work with load square deflection method for plate girder with non-compact flange

Model	=	c/	Pcr (P.W.)	Pcr (L.S.D.M)
NC1	200	8.39286	46.8	51
NC2	200	9.79167	38.9	44
NC1	166	8.35714	48.5	55
NC2	166	9.75	40.5	49
NC1	142	8.32143	51.8	55
NC2	142	9.70833	52.3	51

The presence of initial geometric imperfections, modeled as a percentage of the web thickness, fundamentally alters the girder's response from a bifurcation problem to a stability limit problem. In a theoretically perfect girder, buckling occurs at the critical load () through bifurcation. However, an imperfect girder begins to deflect immediately upon loading. The initial imperfection introduces an eccentricity to the applied load, creating secondary bending moments (P \ w) from the very start. These secondary moments accelerate the growth of lateral deflection, causing the girder to reach its ultimate capacity (the critical moment) at a load significantly lower than the theoretical elastic critical load. The Spencer technique successfully captures this behavior by using the load-deflection data to extrapolate the theoretical critical load, which is a key strength of the methodology in the context of real-world, imperfect structures.

4. Conclusions

In this study, we successfully computed the critical buckling moments for steel I-girders with initial imperfections by pioneering the integration of the Spencer technique with finite element analysis (ANSYS 20). This novel approach proved to be a robust and precise method for stability analysis, particularly for complex structural geometries where traditional analytical methods are limited. The comprehensive parametric study, which varied web slenderness and flange compactness, yielded several key findings. Specifically, the results confirmed that increasing the web slenderness ratio (/) significantly reduces the critical buckling moment, underscoring the critical role of web geometry in stability. Furthermore, the analysis demonstrated that the Spencer technique, when applied to FEA-derived data, provides a reliable estimation of the critical load that aligns well with established design code requirements (SBC 306 and ECP 205), thus validating the proposed methodology. The study's primary contribution lies in offering a practical, verified numerical tool for structural engineers to assess the stability of I-girders under pure bending, explicitly accounting for initial imperfections. This methodological advancement enables a more rigorous assessment of I-girder stability, providing engineers with the precision needed to meet the stringent requirements of modern design codes, such as SBC 306 and ECP 205, particularly in the presence of manufacturing imperfections.

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6. Appendixes

Appendix A: derivations of Southwell and Spencer techniques

A.1 Using the Southwell method to calculate the buckling load

The Southwell method, while foundational, often exhibits non-linearities in the load-deflection plot, particularly at low and high load ranges, which can lead to inaccuracies in the extrapolated critical load [21]. To overcome these limitations, Spencer and Walker [21] proposed an alternative graphical method, known as the Spencer technique, for determining the elastic critical load (P_{cr}) of plates with minor initial imperfections. This technique is particularly robust as it allows for the use of "pivot points" (data points deemed highly accurate) to linearize the data and improve the extrapolation. The method begins with the fundamental relationship for the deflection of an imperfect plate; Southwell made the following assumption about the column's initial shape: a half-sine wave with an initial imperfection at mid-height equal to w_o as: -

$$w_o = A_o \sin\frac{\pi}{L}x$$

Based on this initial shape, the load-deflection relationship can then be roughly represented as [31-32]: -

$$w_t = w_o + w = \left(\frac{1}{1 - {}^{P}/_{P_{cr}}}\right).w_o$$

Solving in terms of yields.

$$w = \frac{P}{P_{cr}}\left(\frac{w_o}{1 - {}^{P}/_{P_{cr}}}\right)$$

Rearranging further gives.

$$\frac{w}{P} = \frac{1}{P_{cr}}w + \frac{1}{P_{cr}}w_o$$

Where: -

w	Deflections functions of plate
w_o	Initial imperfection functions
w_t	Total deflections of plate

The relationship between w/P and is linear within the elastic range, allowing the critical load to be determined from the inverse slope of the line. Figure (1) illustrates this graphical method, showing how the linear extrapolation of the data points yields the anticipated buckling load at the intercept."

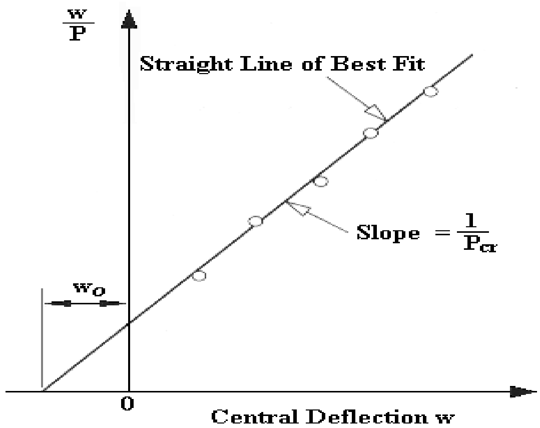

Figure (A1) Southwell plot

6.1 Spencer technique for evaluating critical load

The Spencer technique is based on a modified form of the load-deflection relationship for an imperfect plate, which can be expressed in terms of the total deflection (w) and the applied load (P). The core of the method involves transforming the raw data into two new variables, H1 and H2, which are designed to exhibit a linear relationship when plotted against each other. Any theoretical boundary condition cannot be precisely reproduced in an actual experimental setup, especially for plate tests. In addition, compressive test results may be very susceptible to imperfections [3,12,22,30] and, consequently, to the real physical boundary conditions. Nevertheless, experiments reveal that the Southwell approach occasionally produces nonlinearities that fall into the low-load and higher-load categories. The former may be caused by intrinsic zero errors in the deflection measurements, which can lead to significant mistakes in the apparent critical load , as demonstrated.

Walker [21] demonstrated that a decent approximation that is true well into the post-critical region for rectangular plates across a wide variety of aspect ratios, boundary conditions, and loads as: -

$$[(\frac{w}{t})^2 + (\frac{w_o}{t})^2]^{1/2} = A_c\alpha + B_c\alpha^3$$

$$\alpha = (\frac{P}{P_{cr} - 1} + \frac{w_o}{w})^{1/2}$$

Where:

and and are constants, and t is the plate thickness

With simply supported square plates, uniaxially loaded, is an order of magnitude lower than, so that the second part on the right side of Eq. (6) stays insignificant for = for minor deflections.

Spencer [21] offers an equation that uses “pivot points” to create charts that could aid in linearizing the data. To put it succinctly, a “pivot point” is any actual data point that the researcher determines is accurate enough to justify being included in an analysis as an additional equation. This novel graphical method is suggested for determining critical load for plates with minor flaws after adding pivot points (P*,w*) as:-

$$\frac{1}{P_{cr}}\frac{P^*}{w^*} = \frac{1}{(w^* + w_o)} + \frac{(w^* + 2w_o)}{(A_c t)^2}$$

The constant can be eliminated as.

$$H_2 = (P_{cr}F_1)H_1 - w_o Where H_2 = (Pw^{*2} - P^*w^2)/\Delta\ H1 = (w^{*2} - w^2)/\Delta\Delta = [Pw^*(3 + w^*/w) - P^*w(3 + w/w^*)]F_2$$

$$F_1 = 1 + 3w_o/(w * + w)F_2 = \frac{P[3w * + (w *)^2/w + G *] - P * [3w + w^2/w * + G]}{P[3w * + (w *)^2/w] - P * [3w + w^2/w *]} G * = w_o[2 + 3w * + 2w_o)/w]G = w_o[2 + 3w + 2w_o)/w *]$$

The Spencer technique linearizes the load-deflection data by plotting the derived parameters H2 against H1. This graphical representation, shown in Figure (2), is crucial because the resulting straight line’s slope and intercept allow for the accurate computation of the critical load and the initial imperfection , even with minor flaws in the plate.

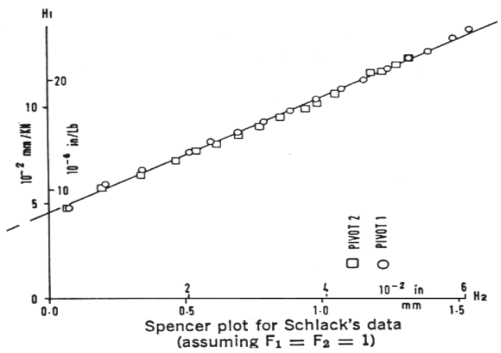

Figure (A2) Spencer technique for predicting the critical load of plates under compression

By applying this technique to the load-deflection data obtained from the Finite Element Analysis (FEA), the elastic critical load can be accurately extrapolated, providing a robust estimate of the critical buckling moment for the imperfect I-girder. This method is superior to the Southwell plot for FEA data, as it minimizes the influence of non-linearities and allows for a more reliable determination of the theoretical elastic buckling capacity.