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Solving Two-Stage Stochastic Programming Problems by Successive Exponential Regression Approximations

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Abstract : There is a variety of technics for solving Two-Stage Stochastic Programming Problems. Recently the two-stage stochastic problem has been solved by the successive quadratic regression approximations. Here, we solve the two-stage stochastic problems by using the exponential regression instead of the quadratic one, then we build the algorithm of successive exponential regression approximations (SERA). Successive exponential regression approximations (SERA) procedure has been improved to solve two-stage stochastic problems by using the same technic of successive quadratic regression approximations where we replace the exponential regression function instead the quadratic regression function in which is replaced instead the expected recourse function of second stage problem which is hard to evaluate numerically, then we compute exponential regression. So the algorithm is used to solve large-scale of multi-stage problems. Also, the new algorithm is solving two-stage problem by using exponential regression approximations which is convergent.

Key Words: Two-stage stochastic problem, exponential regression approximations, quadratic regression approximations.

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حل مشاكل البرمجة العشوائية ذات المرحلتين باستخدام تقديرات الانحدار الأسي المتتالي

ناصر بن عائض الرشيدي

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مستخلص البحث: توجد مجموعة متنوعة من التقنيات لحل مسائل البرمجة العشوائية ذات المرحلتين.مؤخرًا يتم حل مشكلة البرمجة العشوائية ذات المرحلتين من خلال تقريب الانحدار التربيعي المتتالي. هنا قمنا بحل المسائل العشوائية ذات المرحلتين باستخدام الانحدار الأسي بدلاً من الانحدار التربيعي، ثم قمنا ببناء خوارزمية تقريب الانحدار الأسي المتتالي (SERA). تم تحسين إجراء تقريب الانحدار الأسي المتتالي (SERA) لحل المشكلات العشوائية ذات المرحلتين باستخدام نفس تقنية تقريب الانحدار التربيعي المتتالي حيث نستبدل دالة الانحدار الأسي بدلاً من دالة الانحدار التربيعية التي يتم فيها استبدال دالة الرجوع المتوقعة لـ مشكلة المرحلة الثانية والتي يصعب تقييمها عدديا، ثم نقوم بحساب الانحدار الأسي. لذلك يتم استخدام الخوارزمية لحل مشاكل واسعة النطاق متعددة المراحل. كما تقوم الخوارزمية الجديدة بحل مسألة ذات مرحلتين باستخدام تقريب الانحدار الأسي المتقارب.

كلمات مفتاحية: مشكلة العشوائية ذات المرحلتين، تقريب الانحدارالأسي ، تقريب الانحدار التربيعي.

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1. Introduction

Stochastic programming, or called optimization under uncertainty, is an optimization problem formulated mathematically with stochastic systems, where random variable parameters appear in objective functions or in the constraints. Uncertainty is dealt with random parameters in objective or constraints, or in both. (Prekopa,1995) shows a numerical example of a large-scale size of uncertainty problem.

Dynamic Stochastic programming models or static models are decision making problems where the equations are stochastics, (Prekopa, 1995) use a model where some or all of the parameters are random by considering of joint distribution function, for further details see (Prekopa, 1995; Deak, 2001; Deak, 2004; Deak, 2006; Deak, 2011; Shapiro, Dentcheva and Ruszczynski, 2009). Two stage stochastic problem see (Nasser Alreshidi et al., 2020; Rashid Nawaz et al., 2020) they show decomposition method which give quickly convergent and and encouraging results. Convergence of Krasnoselskii–Mann for more details see (Shah, 2022; Nawaz, 2020).

There are many real applications of two-stage models that done in many fields of DM such as Accident prediction models (Chao Wanget et al., 2011). Transportation problem (Hrabec et al., 2015). Outages of power plants (Cot´e and Laughton, 1982). Food supply chain (Bryndis Stefansdottir and Mar- tin Grunow 2018). Portfolio optimization (Nasser Alreshidi et al., 2020). Energy models (Jo˜ao Soares et al., 2017). Airline network (Yang T.H., 2010). Staffing and Scheduling (Kibaek Kim and Sanjay Mehrotra., 2015). Biomass supply chain networks (Maria Aranguren et al., 2021). Water resources problems (Wang and Huang., 2015). Milk production problems (Yalcin and Stott., 2000). Risk (Zimmerman and Carter., 2003). Two-stage stochastic programming with recourse is the most important and most used model in stochastic programming (Prekopa, 1995; Bryndis Stefansdottir and Martin Grunow, 2018). Recently Deak developed a heuristic algorithm, this procedure called successive regression approximations or SRA that is for solving the two-stage and probabilistic stochastic programming problems. The expected recourse function of the second stage problem (A. Ruszczynski and A. Shapiro, 2003), frequently cannot be evaluated accurately but some Monte Carlo techniques can compute them. The algorithm is based on replacing the expected recourse function, which is numerically hard to be solve by the regression function then solving this problem by this heuristic technique, see (Deak, 2001; Deak, 2004; Deak, 2006; Deak, 2011). Deak describes the SRA algorithm for tow stage stochastic programming problem as following. Two-stage stochastic programming with expected recourse:

$$\min c^T \boldsymbol{x} + Q(\boldsymbol{x}) \text{ subject to } A\boldsymbol{x} = b,\ x \geq 0 \qquad (1)$$

Where $Q(\boldsymbol{x}) = E(q(\boldsymbol{x},\xi) = E(min_y q^T y | W_y = \xi - T\boldsymbol{x}, y \geq 0)$ and the vector $\boldsymbol{x} \in R^{n_1}$and $y \in R^{n_2}$are denoted for first-stage decisions and the second-stage decision variable, respectively. All the matrices here are deterministic A, T, W and the dimensions are $m_1 \times n_1$, $m_2 \times n_1$, $m_1 \times n_2$ respectively, for the other vectors are deterministic $b \in R^{m_1}$ and $q \in R^{n_2}$ except $\xi \in R^{m_1}$ the righthand side vector is random. In the SRA algorithm, he assumed [5], that ξ is uncertain with normal distribution and the problem has complete recourse to guarantee that the second stage problem is feasible which, means that for any x and any ξ there exists y feasible solution and the second stage linear programming problem has a finite optimal solution, which means $\forall x$ x, ξ the $q(x,\xi)$ is less than ∞. The difficult part is to compute the expected recourse function because of the multidimensional integral but it is easy to give an unbiased estimate of it. Let $\xi_1, \xi_2, \dots, \xi_k$ to be independent samples from distribution of the random ξ that is for each point $\boldsymbol{x_i}$ then

$q_i = \frac{1}{k}\sum_{i=1}^{k} q(\boldsymbol{x}_i, \xi_i)$ is an unbiased estimate of the expected recourse function $Q(\boldsymbol{x})$ and ξ are independent samples. The SRA algorithm computes this estimated function value and constructs a quadratic approximation based on q_i.

To start the SRA algorithm, we need to make random initial points x_i and compute for each point of x_i an unbiased estimate q_i of $Q(\boldsymbol{x}_i)$which is linear programming problems that is $q_i \sim Q(x_i)$. Then, we have the set $S_k = \{x_i, q_i\}_{i=1}^{k}$ and the quadratic regression function of this form:

$$q_k(\boldsymbol{x}) = \boldsymbol{x}' D_k \boldsymbol{x} + b'_k + c_k$$

is replaced instead the expected recourse function $Q(\boldsymbol{x})$ which is hard to evaluate numerically, Where $\boldsymbol{D}_k$ is a assumed to be symmetric matrices and $\boldsymbol{D}_k$, $\boldsymbol{b}_k$, c_k are unknown parameters can be computed from the optimization problem:

$$\min_{\boldsymbol{D}_k, \boldsymbol{b}_k, c_k} = \sum_{i=1}^{k} [q_i - (x'_i \boldsymbol{D}_k \boldsymbol{x}_i + \boldsymbol{b}'_k \boldsymbol{x}_i + c_k)]^2 \qquad (2)$$

by the first order necessary conditions of (2), the solutions for the unknown parameters are giving by:

$$\sum_{i=1}^{k} [q_i - (\boldsymbol{x}'_i \boldsymbol{D}_k \boldsymbol{x}_i + \boldsymbol{b}'_k x_i + c_k)] = 0$$

$$\sum_{i=1}^{k} [q_i - (\boldsymbol{x}'_i \boldsymbol{D}_k \boldsymbol{x}_i + \boldsymbol{b}'_k x_i + c_k)] \boldsymbol{x}_{im} = 0,\ m = 1, \dots, n.$$

$$\sum_{i=1}^{k} [q_i - (\boldsymbol{x}'_i \boldsymbol{D}_k \boldsymbol{x}_i + \boldsymbol{b}'_k x_i + c_k)] \boldsymbol{x}_{im} \boldsymbol{x}_{il} = 0, l = 1, \dots, n,$$

where x_{im} is the mth component of the vector x_i. Furthermore, we can rewrite the system (2-2)

$$\boldsymbol{M}\Lambda = \boldsymbol{m}, \quad \Lambda = \boldsymbol{M}^{-1}\boldsymbol{m}, \qquad (3)$$

Where $\Lambda' = (d_{11}, d_{12}, \dots, d_{1n}, d_{22}, \dots, d_{2n}, d_{33}, \dots, d_{nn}, d_1, \dots, d_n, \ c)$ and the

$\mathbf{m}' = (m_{2,11}, m_{2,12}, \dots, m_{2,1n}, m_{2,12}, m_{2,22}, \dots, m_{2,nn}, m_{1,1}, m_{1,2}, \dots, m_{1,n}, m_0)$ and the component of **m** are:

$m_0 = \frac{1}{k}\sum_{i=1}^{k} q_i$, $m_{1,m} = \frac{1}{k}\sum_{i=1}^{k} q_i\, x_{im}$, $m_{2,ml} = \frac{1}{k}\sum_{i=1}^{k} q_i\, x_{im} x_{il}$,

And the elements of the matrix M are defined as:

$M_{0,m} = \frac{1}{k}\sum_{i=1}^{k} x_{im}^0 = 1$, $M_{1,m} = \frac{1}{k}\sum_{i=1}^{k} x_{im}$, $M_{2,ml} = \frac{1}{k}\sum_{i=1}^{k} x_{im} x_{il}$, $M_{3,mlr} = \frac{1}{k}\sum_{i=1}^{k} x_{im} x_{il} x_{ir}$ and $M_{4,mlrs} = \frac{1}{k}\sum_{i=1}^{k} x_{im} x_{il} x_{ir} x_{is}$

"These notations used to describe the matrix **M** and by solving the system (3), so the solution of problem (2) will obtain.

The SRA algorithm for two-stage problem which introduced by (Deak, 2004) is giving as following:

0. [Initialization.] Set the iteration counter to the number k of points and compute $q_i \sim Q(\boldsymbol{x}_i)$ and $S_k = \{\boldsymbol{x}_i, q_i\}_{i=1}^{k}$.
1. Compute the coefficients of $\boldsymbol{D}_k$, $\boldsymbol{b}_k$ and c_k of the quadratic regression function

$$q_k(x) = \boldsymbol{x}' \boldsymbol{D}_k x + \boldsymbol{b}'_k + c_k \qquad (4)$$

from S_k by solving the minimization problem (2).

2. Replace the original first stage problem with the

following approximate one:

$$\min_{\boldsymbol{x}} = c'\boldsymbol{x} + q_k(\boldsymbol{x}) \quad (5)$$
$$A\boldsymbol{x} \leq b$$
$$x \geq 0$$

and denote its optimal solution by $\boldsymbol{x}_k$.

3. If $\boldsymbol{x}_k$ is "good enough" then STOP, otherwise compute $q_k \sim Q(\boldsymbol{x}_k)$, let $S_{k+1} = S_k \cup \{\boldsymbol{x}_k, q_k\}$ increase$k := k+1$ and go back to Step 1" (Deak , 2004), the above algorithm was described and approved theoretically by (Deak, 2004).

The most recent development in the solution of two-stage model with probabilistic constraint is a heuristic approach (successive regression approximations (SRA) proposed by (Deak, 2003) for medium-size problems, which is extended for large scale problem with one hundred decision variables in the and first-stage with 120 dimensional normally distributed ξ in the second stage problem (Deak, 2011), where he claims that the computational test indicates that the method is working. However, no theoretical proof of the SRA method exists but the performance has been efficient for more details see (Deak, 2002; Deak, 2003; Deak, 2006). $Q(\boldsymbol{x}) = E(q(x,\xi) = E(min_y q^T y | W_y = \xi - Tx, y \geq 0)$ where q is a random variable and we assume that the matrices T and W are deterministic. The main difficult computationally is computing the value of the expected recourse when the being multidimensional integral is hard to calculate the expected value. It is easily computed for any x and ξ_i by unbiased estimate of it:

2. Computing a Least Squares-Regression for Exponential Approximation

Assume that we have k distinct points, for instance$(x_1, y_{1)}, \dots (x_k, y_{k)})$ and we need to interpolate a function $g_i = g(x_i)$ such that $g_i = y_i$ for all $i = 1, \dots, k$ Consider we need to an interpolation for these points that satisfy the exponential function, i.e.,

$$y_i = \alpha\, e^{\beta x_i}, \text{for } \ i = 1, 2, \dots, k \quad (6)$$

where α and β are unknown constants. In order to give the best approximation for the function g_i we shall solve the least square problem (L^2 minimum norm), that is,

$$\min_{\alpha,\beta} \textstyle\sum_{i=1}^{n} [g_i - y_i]^2 \quad (7)$$

Apply the natural logarithm for both sides of Eq. (6) then we obtain

$$lny_i = \ln\left(\alpha\, e^{\beta x_i}\right) = ln\alpha + \beta x_i$$

Let $f_i = lny_i$ and $\gamma = ln\alpha$, then the least square problem (7) can be transformed into

$$\min_{\alpha,\beta} \textstyle\sum_{i=1}^{n} [f_i - (\gamma + \beta x_i]^2 \quad (8)$$

Hence, we will solve the minimization problem (8) that is associated to the pints (x_i, f_i) for $i = 1, \dots, k$, then, we will obtain the solution of the minimization problem (7) by using $\alpha = \exp(\gamma)$. By differentiating (8) with respect to γ and β, respectively, and putting the derivatives equal to zero. Then we have

$$\sum_{i=1}^{n}[f_i - (\gamma + \beta x_i] = 0$$

$$\sum_{i=1}^{n} x_i[f_i - (\gamma + \beta x_i] = 0$$

By solving the above system in the unknowns constants γ and β following the same technique as given by [4], consider

$m_0 = \frac{1}{k}\sum_{i=1}^{k} f_i,$ $\quad m_1 = \frac{1}{k}\sum_{i=1}^{k} f_i\, x_i,$

$M_0 = \frac{1}{k}\sum_{i=1}^{k} x_i^0 = 1,$ $\quad M_1 = \frac{1}{k}\sum_{i=1}^{k} x_i,$ $\quad M_2 = \frac{1}{k}\sum_{i=1}^{k} x_i^2,$

Then, we can rewrite the system (3–3) as follows $\beta M_2 + \gamma M_1 = m_1,\quad \beta M_1 + \gamma M_0 = m_0,$ that is

$$\begin{pmatrix} M_2 & M_1 \\ M_1 & M_0 \end{pmatrix}\begin{pmatrix} \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} m_1 \\ m_0 \end{pmatrix}.$$

Thus, we obtain $\beta = \frac{-m_0 M_1 + m_1}{M_2 - M_1^2}$, and $\gamma = \frac{-m_0 M_2 - m_1 M_1}{M_2 - M_1^2}$.

Furthermore, the solution of the minimization problem (7) is given by

$$\beta = \frac{k\sum_{i=1}^{k} x_i lny_i - \sum_{i=1}^{k}\sum_{i=1}^{k} x_i lny_i}{k\sum_{i=1}^{k} x_i^2 - \left(\sum_{i=1}^{k} x_i\right)^2},\ \text{and}$$

$$\gamma = exp\left[\frac{-\sum_{i=1}^{k} x_i \sum_{i=1}^{k} x_i\, lny_i + \sum_{i=1}^{k} lny_i \sum_{i=1}^{k} x_i^2}{k\sum_{i=1}^{k} x_i^2 - \left(\sum_{i=1}^{k} x_i\right)^2}\right]$$

3. Multidimensional SERA

Consider we have $\boldsymbol{k}$ distinct points $\boldsymbol{x_i}, \mathbf{b} \in \mathbf{R^n}$ for $i = 1, 2, \dots, k$. We need an interpolate these points such that satisfies the exponential function

$$y_i = \alpha e^{\boldsymbol{b}'x},\ \text{for } i = 1, 2, \dots, k \qquad (9)$$

where α and b are unknown constants. Similarly as for the previous simple case, by taking the logarithm for both sides of Eq. (9). Assume that $f_i = lny_i$ and $\gamma = ln\alpha$, then by solving the following minimization problem we can compute the L^2 minimum norm, that is

$$\min_{\gamma, \beta} \sum_{i=1}^{n}[\boldsymbol{f}_i - (\gamma + \boldsymbol{b}'\boldsymbol{x}]^2 \qquad (10)$$

for the unknown constant γ and b. By differentiating (10) with respect to γ and b, respectively. Then we obtain $\sum_{i=1}^{n}[\boldsymbol{f}_i - (\gamma + \boldsymbol{b}'\boldsymbol{x}] = 0,$

$\sum_{i=1}^{n} x_{ij}[\boldsymbol{f}_i - (\gamma + \boldsymbol{b}'\boldsymbol{x}] = 0,$ for $j = 1, 2, \dots, n,$

where x_{ij} is the $j-th$ component of the vector x_i. Following [5], we consider the following notations,

$m_0 = \frac{1}{k}\sum_{i=1}^{k}\boldsymbol{f}_i,\quad m_{1,j} = \frac{1}{k}\sum_{i=1}^{k} x_{ij}\boldsymbol{f}_i,$

$M_{0,j} = \frac{1}{k}\sum_{i=1}^{k} x_{ij}^0 = 1,\quad M_{1,j} = \frac{1}{k}\sum_{i=1}^{k} x_{ij},$ and $M_{2,jl} = \frac{1}{k}\sum_{i=1}^{k} x_{ij}x_{il},$

Furthermore we can rewrite the above system as follows

$$\boldsymbol{M}\Lambda = \boldsymbol{m}, \qquad (11)$$

where Λ is the vector of the unknown constants, $\Lambda' = (b_1, \dots, b_n)$, and and the vector m and the matrix and $\boldsymbol{M}$ are defined by

$$\boldsymbol{m} = (m_{1,1}, \dots, m_{1,n}, m_0)$$

$$\boldsymbol{M} = \begin{pmatrix} M_{2,11} & \cdots & M_{2,1n} & M_{1,1} \\ \vdots & \ddots & \vdots & \vdots \\ M_{2,n1} & \cdots & M_{2,nn} & M_{1,n} \\ M_{1,1} & \cdots & M_{1,n} & M_0 \end{pmatrix}$$

obtain the solution of the minimization problem (10).

4. SERA Algorithm for the Two-Stage Problem

The two-stage programming with recourse can be written as:

$$\min c^T \boldsymbol{x} + Q(\boldsymbol{x}) \text{ subject to } A\boldsymbol{x} = b,\ x \geq 0 \qquad (12)$$

where the expected recourse function Q(x) can be giving as $Q(\boldsymbol{x}) = E(q(\boldsymbol{x}, \xi) = E(min_y q^T y | W_y = \xi - T\boldsymbol{x}, y \geq 0)$ where q is a random variable and we assume that the matrices T and W are deterministic. The main difficult computationally is computing the value of the expected recourse when the being multidimensional integral is hard to calculate the expected value. It is easily computed for any x and $\xi_i$by unbiased estimate of it:

$$q_i = q(\boldsymbol{x}, \xi_i) = min\ q^T y \quad \text{s.t} \quad T\boldsymbol{x} + \boldsymbol{W}y = \xi_i,\ \ y \geq 0 \qquad (13)$$

We can easily compute the expected recourse $E(q(\boldsymbol{x}, \xi)) =$ by given an unbiased estimate of it. Let $\xi_i$to be independent sample with the random variable ξ, for each $\boldsymbol{x}_i$ then $q_i = q(\boldsymbol{x}_i, \xi_i)$ and $q_i \sim Q(\boldsymbol{x}_i)$ for $i = 1, 2, \dots, k$. In this case: $q_i = \frac{1}{k}\sum_{i=1}^{k} q(\boldsymbol{x}_i, \xi_i)$ are unbiased estimates and independent samples of ξ.

The expected recourse function is replaced by a least squares-regression for exponential approximation regression function of the form:

$$q_k = \alpha_k e^{\boldsymbol{b'}_k x} \qquad (14)$$

To find the solution for the unknown α_k and b_k, by solving the following minimization problem, we can compute the L^2 minimum norm, that is

$$\min_{\gamma, b} \sum_{i=1}^{n} [q_i - q_k(\boldsymbol{x}_i)]^2 \qquad (15)$$

Since this is a minimization problem then the first order necessary conditions of the above problem give the solution of the unknown α_k and b_k:

$$\sum_{i=1}^{k} [q_i - (\gamma_k + \boldsymbol{b}' x_i] = 0,$$

$$\sum_{i=1}^{k} x_{ij} [q_i - (\gamma_k + \boldsymbol{b}' x_i] = 0,\ j = 1, 2, \dots, k.$$

where $\boldsymbol{x}_{ij}$ is the $j - th$ component of the vector $\boldsymbol{x}_i$ and $\gamma = ln\alpha$ for more details see previous section.

For starting our algorithm we need to generate random k points of $\boldsymbol{x}_i$ and calculate q_i for these points. Thus giving the set $S_k = \{\boldsymbol{x}_i, q_i\}_{i=1}^{k}$

0. [start] let the iteration being with the number k of points in S_k.
1. Then, compute the following coefficient b_k and α_k of exponential regression function

$$q_k(\boldsymbol{x}) = \alpha_k e^{\boldsymbol{b'}_k x} \qquad (16)$$

from S_k by solving the minimization problem (15).

2. Replace first-stage problem by the following approximate problem:

$$\min_{\boldsymbol{x}} = c'\boldsymbol{x} + q_k(\boldsymbol{x}) \quad (17)$$

$$A\boldsymbol{x} \leq b$$

$$\boldsymbol{x} \geq 0$$

And the solution denoted by $\boldsymbol{x}_k$

3. If $\boldsymbol{x}_k$ is "optimal", then the solution found, otherwise generate a sample for ξ_k and calculate $q_k \sim Q(\boldsymbol{x}_k)$ and add it to the previous set that is, $S_{k+1} = S_k \cup \{\boldsymbol{x}_k, q_k\}$ by increasing the number f iteration $k := k + 1$ and go back to the second step.

6. Conclusion

Stochastic programming has gained a major of optimization for modeling uncertainties in mathematical optimization problems. Two-stage stochastic programming with random is dealing the problem under uncertainty in models, and use optimization concepts optimization along with statistics and probability. stochastic programming continues develop a huge of algorithm and theoretical by researchers and scientists. In this paper, we build the algorithm of two stage problem which we name it successive exponential regression approximations (SERA) to solve the two-stage stochastic programming for both one-dimension and multidimensional. The algorithm for solving a two-stage model with probabilistic constraint (successive exponential regression approximations (SERA)) was proposed based on replacing the expected recourse function, which is numerically hard to be solve by the regression function then solving this problem by this technique. So by this idea we can solve any two stage Stochastic programming. For future work we will use real application data to with successive exponential regression approximations (SERA).

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